INFO - H - 501

Pattern recognition and image analysis

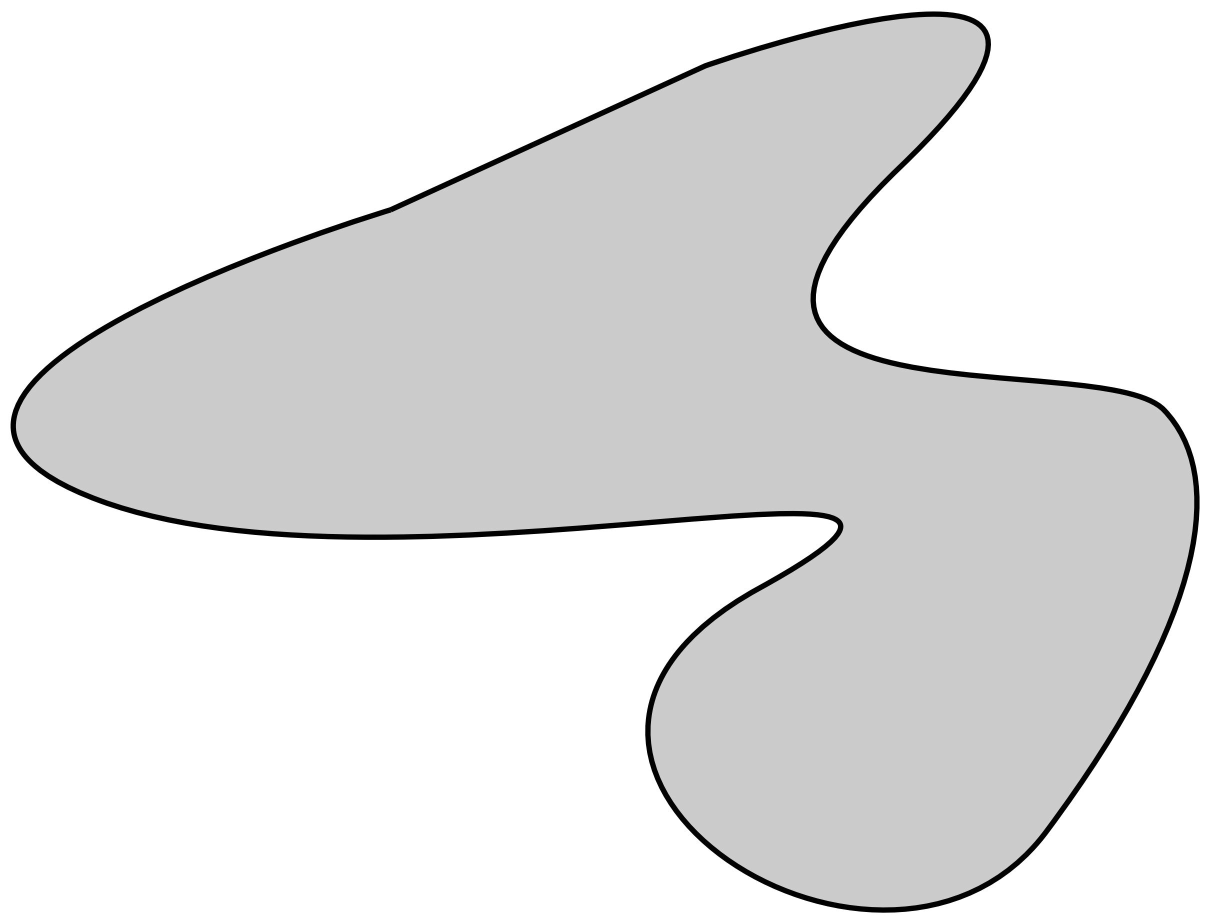
Shape description

Shape description

- chain coding [DIP] p.484
- polygonal approximation [DIP] p.486
- spectral description (intensity) [IPH] p482
- 2D shape descriptors (contours) [DIP] p.499 , [IPH] p490
- 2D shape descriptors (region) area, euler, elong,... [IPAMV] p222
- moments [DIP] p514
- Minkowski fractal dimension [IPH] p442

Shape descriptors

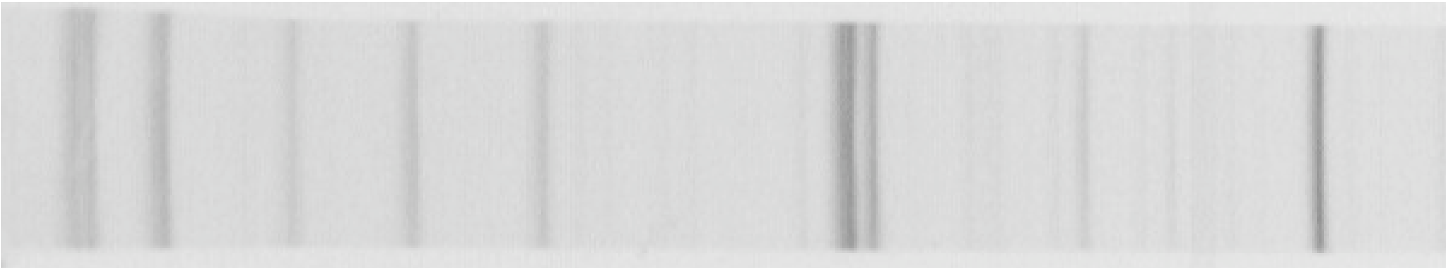
- intensity
- contours
- topology
- texture



Intensity

- Optical Density (O.D.)

$$O.D. = \log_{10}(\frac{I}{I_0})$$



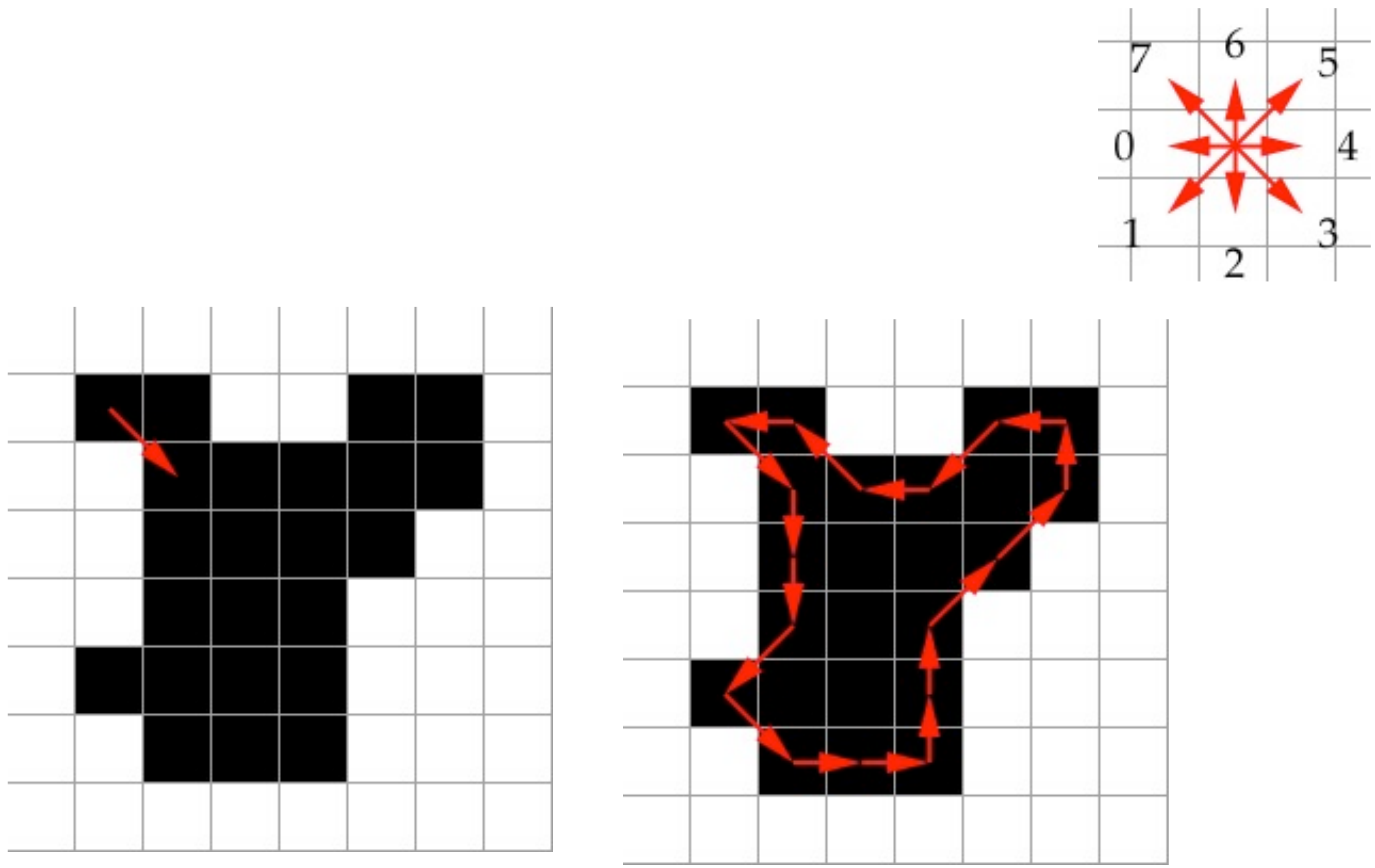
- statistical measurs : mean, standard deviation,...

Shape

- area
- perimeter
- convexe area
- bounding box
- convexity
- shape factor
- major and minor axis

Shape

- chain coding



Shape

- chain code centroid

$$g_x = \frac{\sum_i (x_i + x_{i-1})^2 (y_i - y_{i-1})}{Area}$$

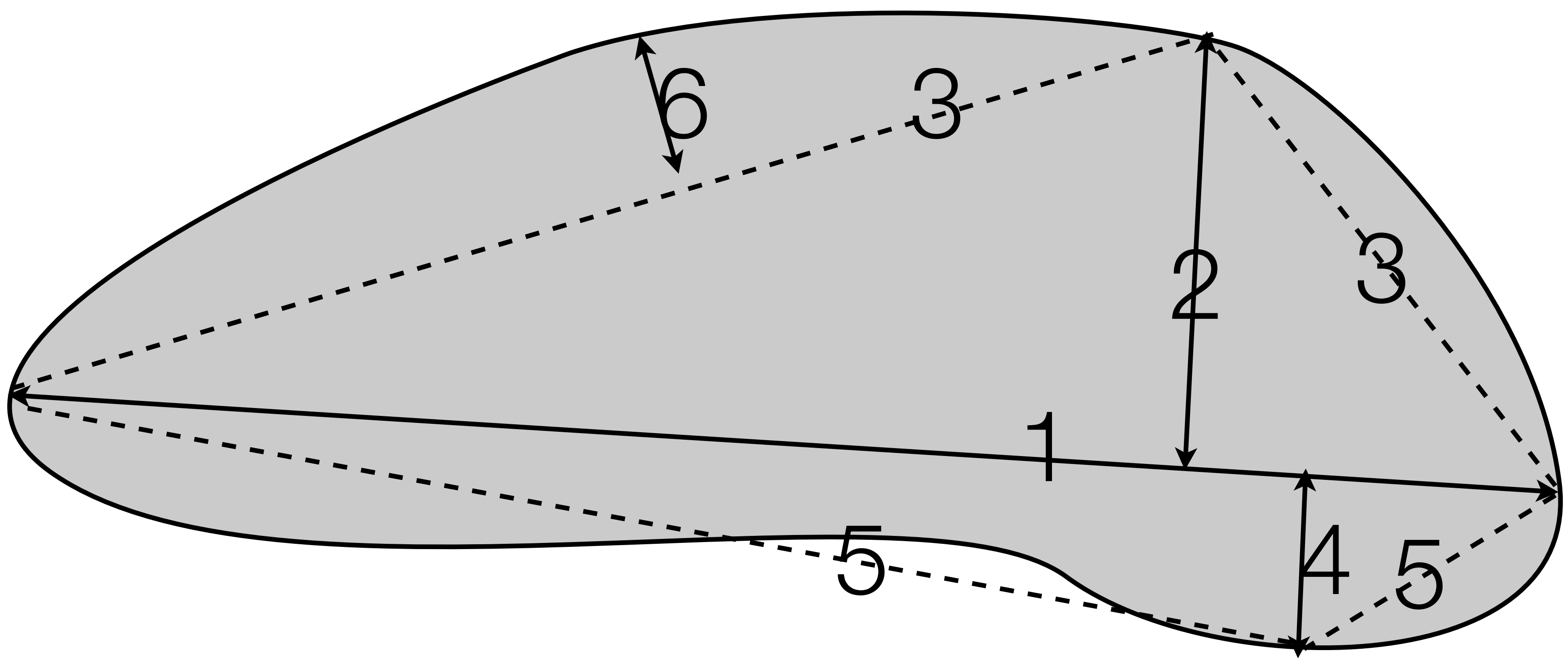
$$g_y = \frac{\sum_i (y_i + y_{i-1})^2 (x_i - x_{i-1})}{Area}$$

- with

$$Area = \frac{\sum_i (x_i + x_{i-1})(y_i - y_{i-1})}{2}$$

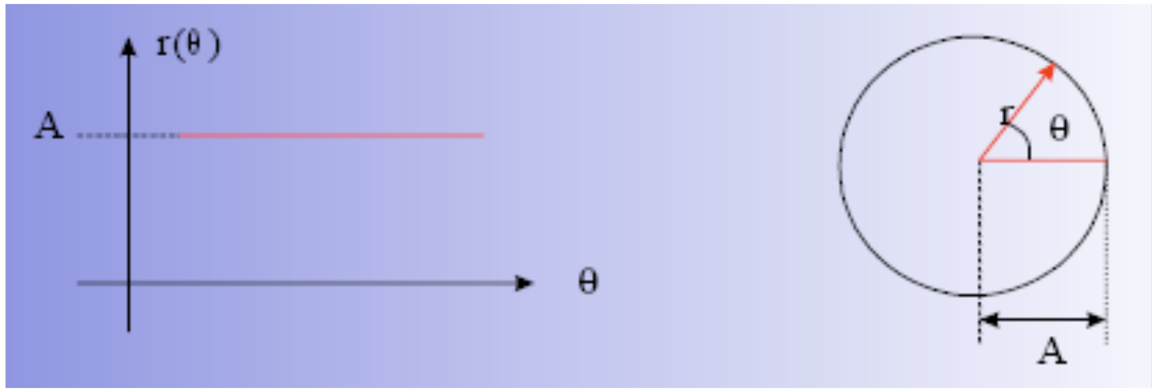
Shape

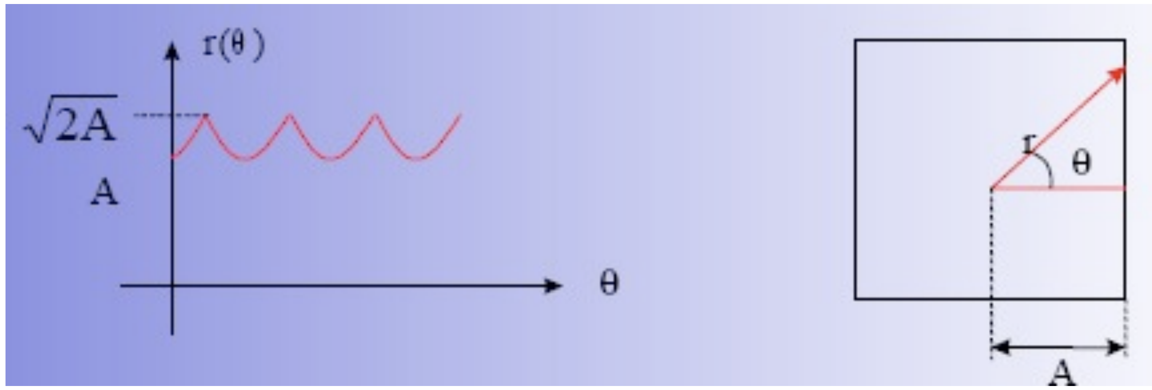
- polygonal approximation



Shape

- contour is unwrapped around shape centroid
- R = f(θ) is periodical
- holomorphic





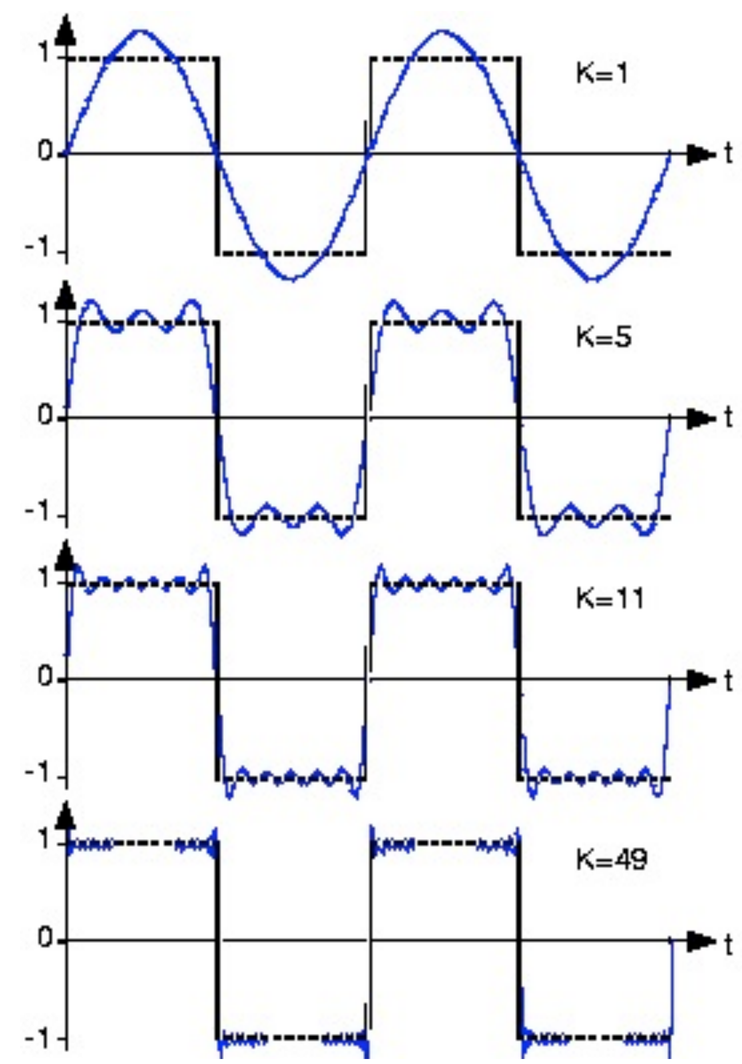
Fourier coefficient of the contour

- periodical function decomposition

$$R(\theta) = A_0 + \sum_{n=1}^{\infty} (A_n \cos n\theta + B_n \sin n\theta)$$

- complex notation

$$R(\theta) = \sum_{n=-\infty}^{\infty} C_n e^{-jn\theta}$$



Fourier coefficient of the contour

- coefficient are:

$$C_n = \frac{1}{2\pi}\int_0^{2\pi} R(\theta)e^{-jn\theta}\,d\theta$$

Fourier coefficient of the contour

- continuous part = mean radius

$$C_0 = \frac{1}{2\pi}\int_0^{2\pi} R(\theta)d\theta$$

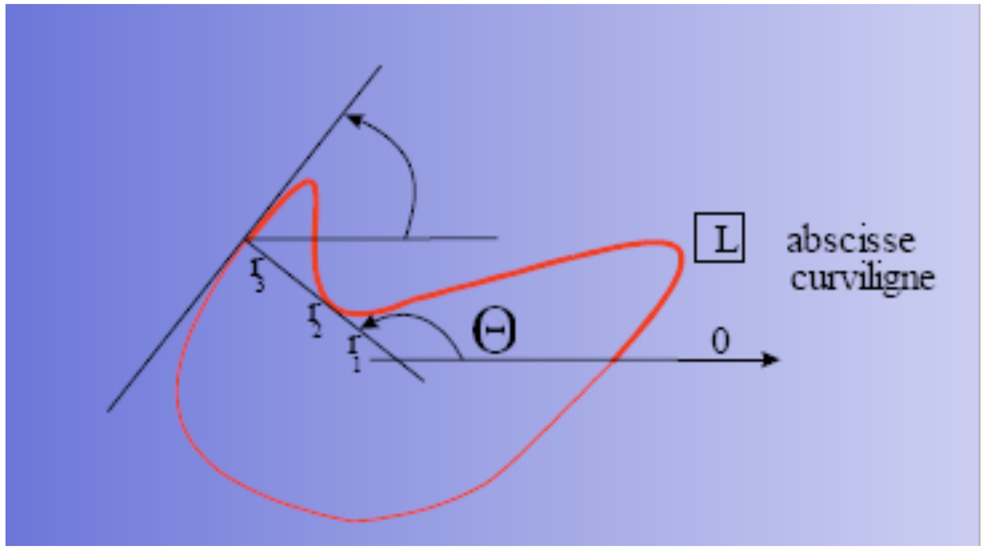
- translation invariant

- NOT rotation invariant !

$$C_n(R(\theta - \alpha)) = C_n(R(\theta))e^{-jn\alpha}$$

Descripteur de bord

- problem : several R for a same angle θ
- Solution : R=f(s) with s the curvilinear abscise
- normalized using perimeter S between 0 and 2π



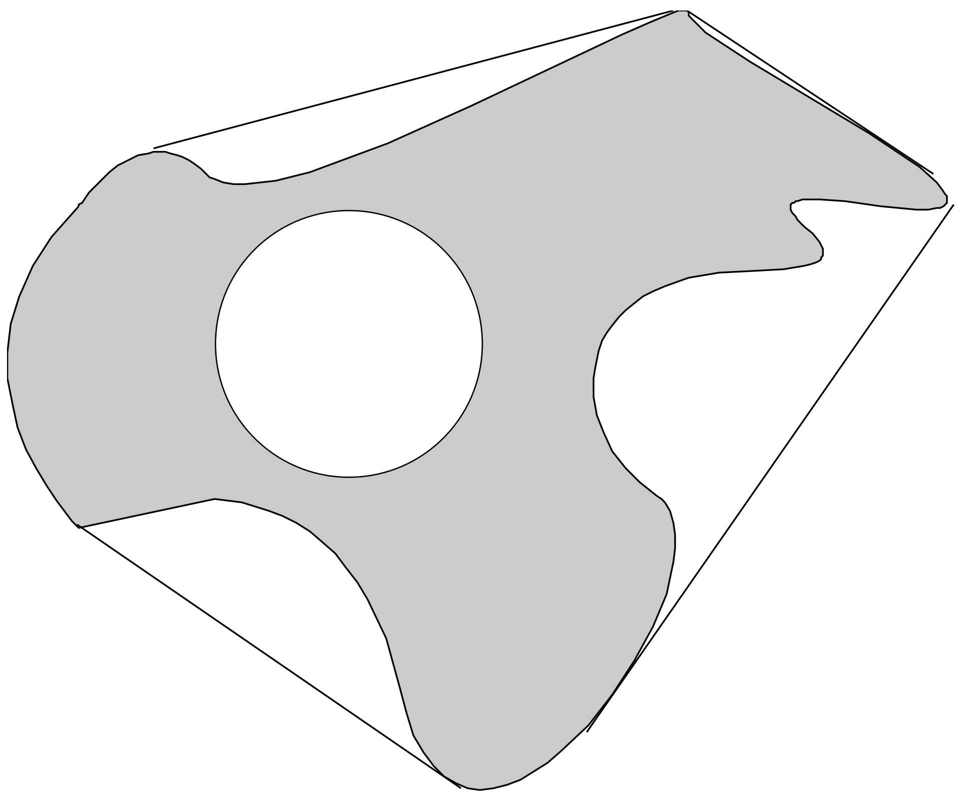
Fourier coefficient of the contour

- Coefficient are computed using:

$$C_n(R(s)) = \frac{1}{2\pi}\int_0^{2\pi} R(s)e^{-jns}ds$$

Shape descriptors

- perimeter L
  - ! √2
- Area S
  - ! fraction of pixels
- Euler nuber = #contours - #holes



Shape descriptors

$$Formfactor = \frac{4\pi Area}{Perimeter}$$

$$Convexity = \frac{ConvexPerimeter}{Perimeter}$$

$$Eq.dia. = \sqrt{\frac{4}{\pi} Area}$$

$$Solidity = \frac{Area}{ConvexArea}$$

$$AspectRatio = \frac{Max\, diameter}{Min\, diameter}$$

Orientation

$$
\begin{aligned}
S_x &= \sum x_i \\
S_y &= \sum y_i \\
S_{xx} &= \sum x_i^2 \\
S_{yy} &= \sum y_i^2 \\
S_{xy} &= \sum x_i y_i
\end{aligned}
$$

$$
\begin{aligned}
M_{xx} &= S_{xx} - \frac{S_x^2}{Area} \\
M_{yy} &= S_{yy} - \frac{S_y^2}{Area} \\
M_{xy} &= S_{xy} - \frac{S_x S_y}{Area}
\end{aligned}
$$

$$
\theta = \tan^{-1}\left\{\frac{M_{xx} - M_{yy} + \sqrt{(M_{xx} - M_{yy})^2 + 4M_{xy}^2}}{2M_{xy}}\right\}
$$

Moments (continuous)

- moments

$$m_{pq} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} x^p y^q f(x,y)\, dx\, dy$$

- centered moments

$$\mu_{pq} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (x - \bar{x})^p (y - \bar{y})^q f(x,y)\, dx\, dy$$

$$\bar{x} = \frac{m_{10}}{m_{00}}, \bar{y} = \frac{m_{01}}{m_{00}}$$

Moments (discret)

- Moments

$$m_{pq} = \sum_{-\infty}^{+\infty} \sum_{-\infty}^{+\infty} x^p y^q f(x,y)$$

$$\mu_{pq} = \sum_{-\infty}^{+\infty} \sum_{-\infty}^{+\infty} (x - \overline{x})^p (y - \overline{y})^q f(x,y)$$

- centered moments

$$\eta_{pq} = \frac{\mu_{pq}}{\mu_{00}^{\gamma}}$$

- normalized centered moment

$$\text{avec } \gamma = \frac{p+q}{2} + 1$$

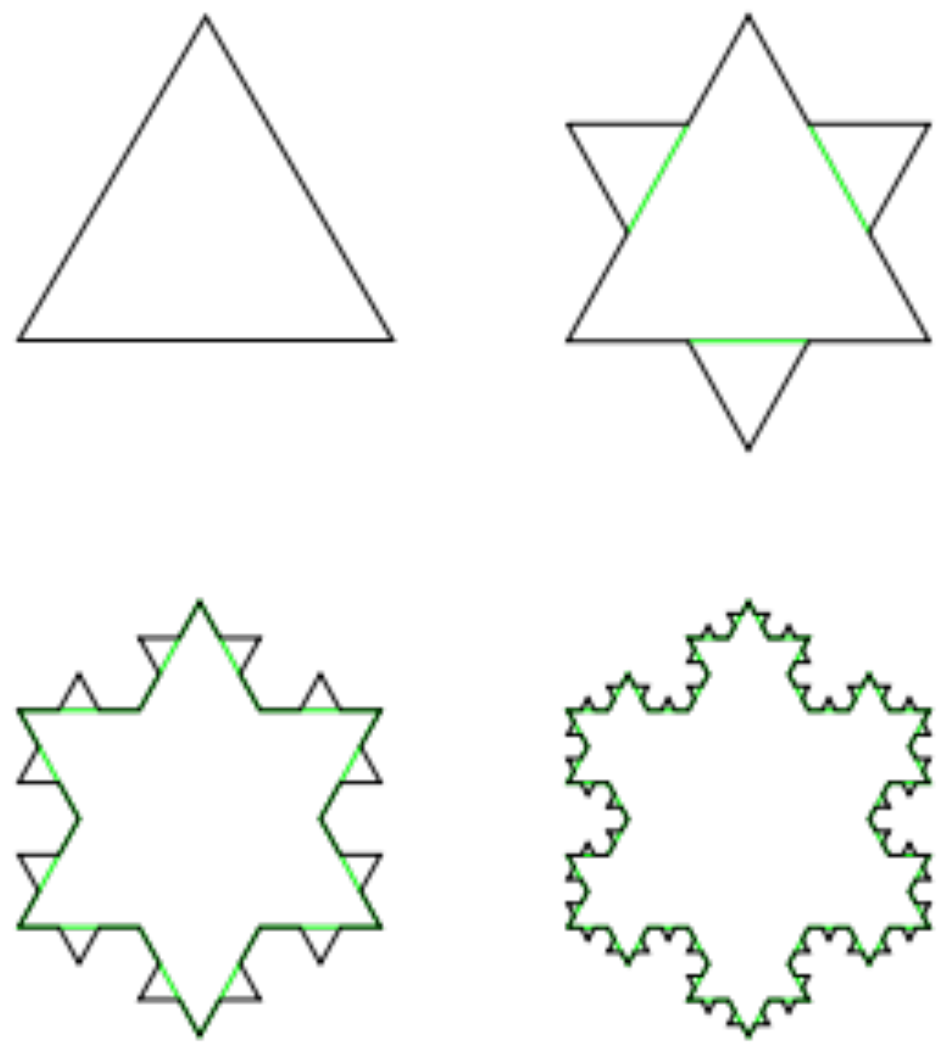
Moments

- Invariants

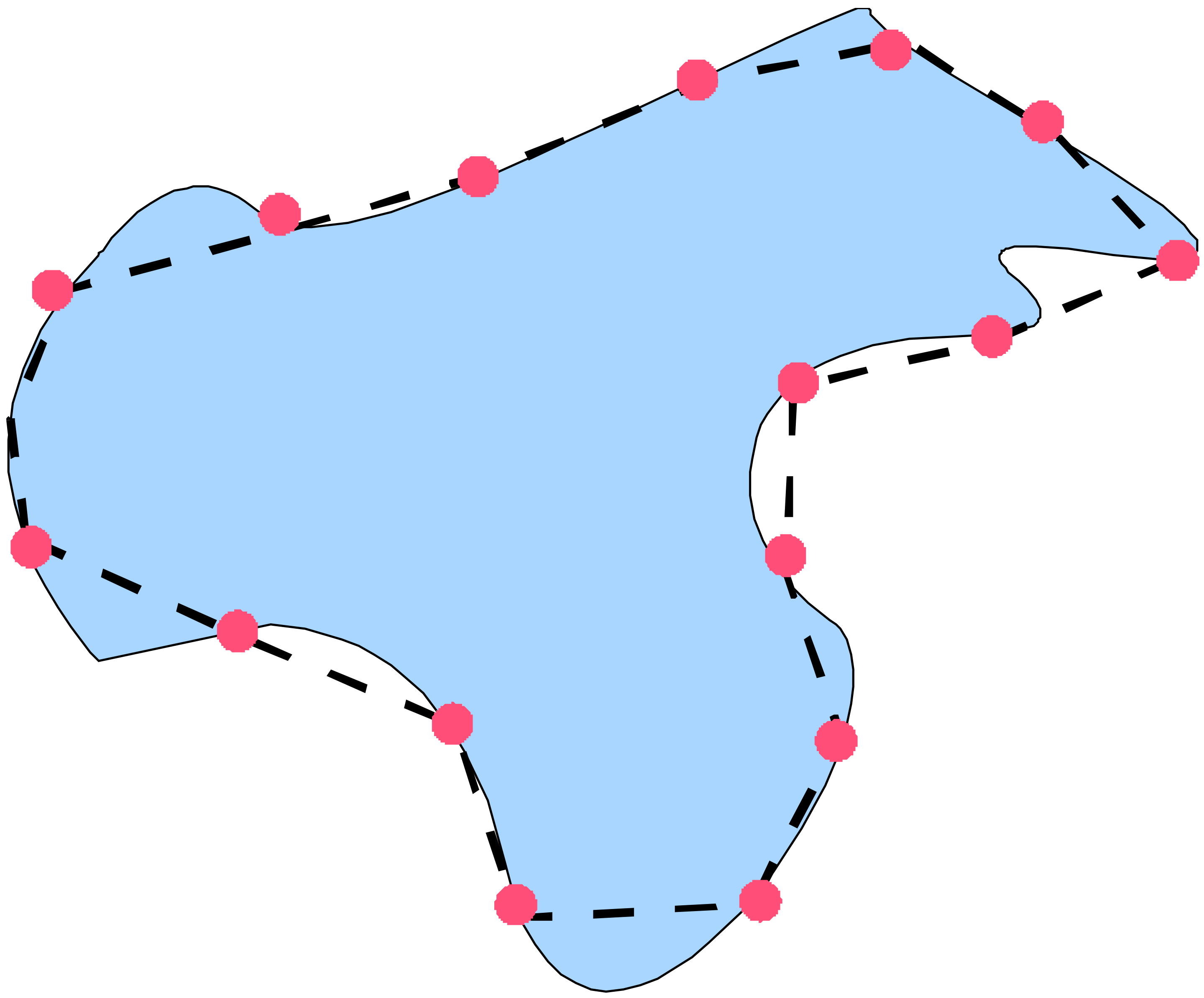
$$
\begin{aligned}
I_1 &= \eta_{20} + \eta_{02} \\
I_2 &= (\eta_{20} - \eta_{02})^2 + (2\eta_{11})^2 \\
I_3 &= (\eta_{30} - 3\eta_{12})^2 + (3\eta_{21} - \eta_{03})^2 \\
I_4 &= (\eta_{30} + \eta_{12})^2 + (\eta_{21} + \eta_{03})^2 \\
I_5 &= (\eta_{30} - 3\eta_{12})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2] + \\
&\quad (3\eta_{21} - \eta_{03})(\eta_{21} + \eta_{03})[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] \\
I_6 &= (\eta_{20} - \eta_{02})[(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] + 4\eta_{11}(\eta_{30} + \eta_{12})(\eta_{21} + \eta_{03}) \\
I_7 &= (3\eta_{21} - \eta_{03})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2] + \\
&\quad (\eta_{30} - 3\eta_{12})(\eta_{21} + \eta_{03})[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2].
\end{aligned}
$$

Fractal measure

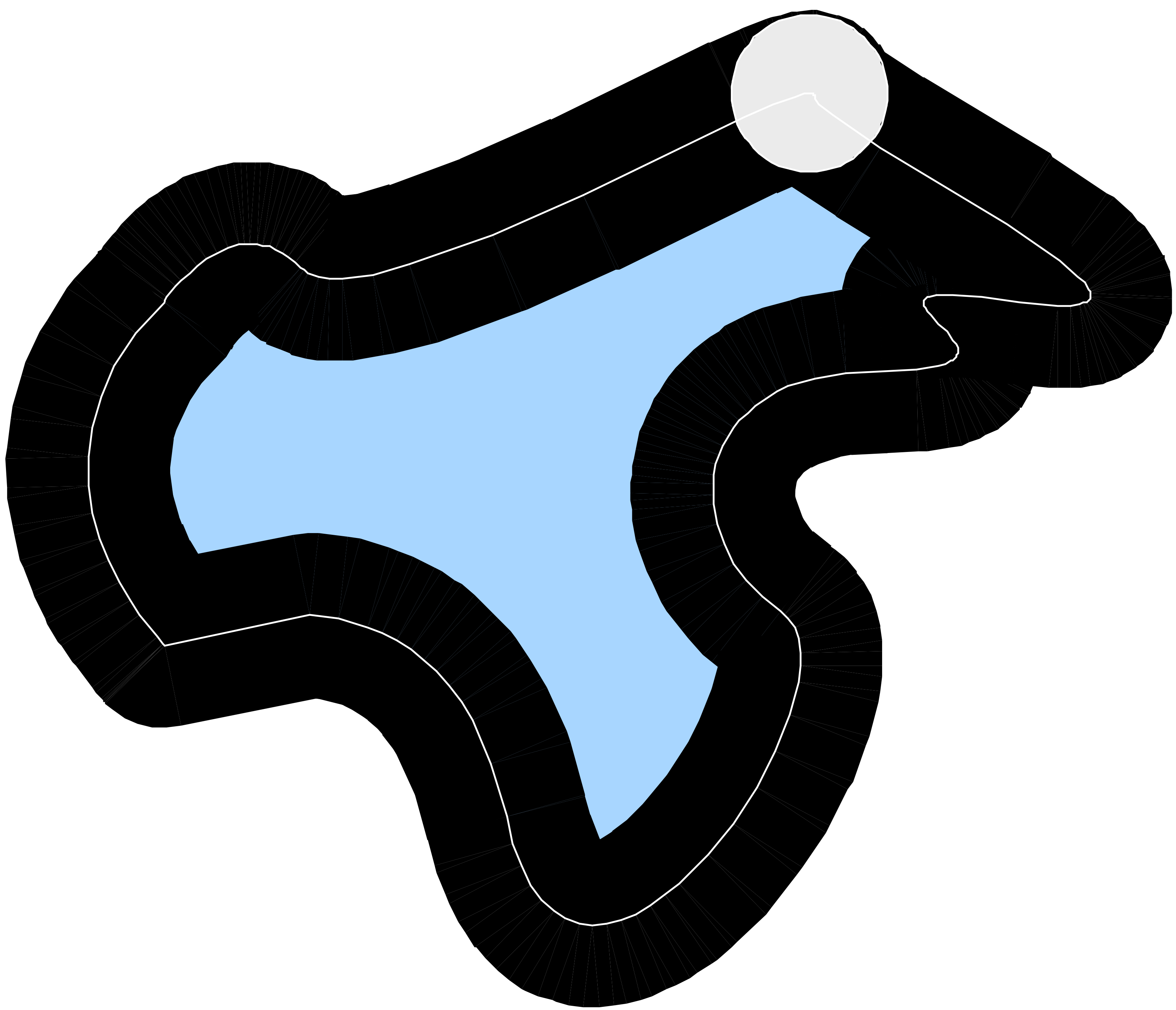
- introduced by Mandelbrot 1967



Fractal measure



Fractal measure



Fractal measure

- perimeter measure depends on unit

$$\log L_2(X,\lambda_i) = f \log \lambda_i$$